# חAmIBIA UחIVERSITY <br> OF SCIEПCE AחD TECHחOLOGY 

## FACULTY OF ENGINEERING AND SPATIAL SCIENCE

DEPARTMENT OF Mining and Process Engineering

| QUALIFICATION : Bachelors of Engineering in Mining Engineering |  |
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| QUALIFICATION CODE: BEMIN | LEVEL: 8 |
| COURSE CODE: OPC 711S | COURSE NAME: OPERATIONS RESEARCH |
| SESSION: JUNE 2022 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SECOND OPPORTUNITY QUESTION PAPER |  |
| :--- | :---: |
| EXAMINER(S) | Lawrence Madziwa |
| MODERATOR: | Dr Mallikarjun Rao Pillalamarry |

## INSTRUCTIONS

1. Answer all questions.
2. Read all the questions carefully before answering.
3. Marks for each questions are indicated at the end of each question.
4. Please ensure that your writing is legible, neat and presentable.

## PERMISSIBLE MATERIALS

1. Examination paper.
2. The mine stores manager is concerned about the long queues of lorries waiting to deliver goods on the mine. Occasionally there are as many as 100 deliveries a week, and in some cases the lorry drivers have had to wait several hours before they can unload at one unloading bay. This has resulted in congestion at the warehouse and complaints from the lorry drivers. You have been asked to make recommendations for improving the situation and have collected delivery data as in the table below.
[20]

| Number of <br> Lorries of <br> arriving per <br> hour | Number <br> hours | Unloading <br> (minutes) | Number of <br> lorries |
| :--- | :--- | :--- | :--- |
| 0 | 7 | $0-20$ | 38 |
| 1 | 10 | $20-40$ | 26 |
| 2 | 8 | $40-60$ | 10 |
| 3 | 8 | $60-80$ | 3 |
| 4 | 5 | $80-100$ | 2 |
| 5 | 2 | $100-120$ | 1 |

Deliveries are allowed between 9 am and 5pm, Monday to Friday. Any lorry that arrive after 5 pm can join the queue awaiting unloading. Unloading crew can work overtime.
a. Explain the conditions which must be satisfied in order to apply the basic single server queuing model ( $\mathrm{M} / \mathrm{M} / 1$ ).
b. Assuming that an $M / M / 1$ model is appropriate, convert the data into suitable information for queuing.
(4)

| Number <br> of Lorries <br> arriving <br> per hour <br> (x) | Number <br> of <br> (fours | xf | Unloading <br> time <br> (minutes) | $\mathbf{y}$ | Number <br> of lorries f | yf |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 7 | 0 | $0-20$ | 10 | 38 | 380 |
| 1 | 10 | 10 | $20-40$ | 30 | 26 | 780 |
| 2 | 8 | 16 | $40-60$ | 50 | 10 | 500 |
| 3 | 8 | 24 | $60-80$ | 70 | 3 | 210 |
| 4 | 5 | 20 | $80-100$ | 90 | 2 | 180 |
| 5 | 2 | 10 | $100-120$ | 110 | 1 | 110 |
|  | 40 | 80 |  |  | 80 | 2160 |

c. Estimate how many lorries, on average are waiting to be unloaded and also the time that a lorry would expect to spend at the warehouse.
At any time the expected number of trucks in the system
d. The unloading bay is currently staffed by two employees who are each paid $\$ 100$ for a 40 hour week, with any overtime being paid at a $4 / 3$ rate. A suggestion has been made that a third person should be employed in the unloading ay which, it has been estimated, would result in saving of seven minutes in the average time to unload a lorry. This, it has been claimed, would not only reduce the lorry waiting time but would also produce a saving in cost to the mine. Analyse this suggestion and make a recommendation. (8)
2. A CONSTRUCTION COMPANY has just made the winning bid of $\$ 5.4$ million to construct a new plant for a major manufacturer. The manufacturer needs the plant to go into operation within a year. Therefore, the contract includes the following provisions:

- A penalty of $\$ 300,000$ if the company has not completed construction by the
deadline 47 weeks from start.
- To provide additional incentive for speedy construction, a bonus of $\$ 150,000$ will be paid to the company if the plant is completed within 40 weeks.
a. The following details pertaining to the project. Construct the network diagram and compute the project completion time.

Table 1: Activities description and predecessors

| Activity | Activity Description | Immediate <br> Predecessors |
| :---: | :--- | :--- |
| A | Excavate | - |
| B | Lay the foundation | A |
| C | Put up the rough wall | B |
| D | Put up the roof | C |
| E | Install the exterior plumbing | C |
| F | Install the interior plumbing | E |
| C | Put up the exterior siding | D |
| H | Do the exterior painting | $\mathrm{E}, \mathrm{G}$ |
| 1 | Do the electrical work | C |
| I | Put up the wallboard | $\mathrm{E}, 1$ |
| $K$ | Install the flooring | J |
| L | Do the interior painting | J |
| M | Install the exterior fixtures | H |
| N | Install the interior fixtures | $\mathrm{K}, \mathrm{L}$ |

Table 2: Activity times

| Activity | Optimistic <br> Estimate <br> $\boldsymbol{o}$ | Most Likely <br> Estimate <br> $\boldsymbol{m}$ | Pessimistic <br> Estimate <br> $\boldsymbol{p}$ |
| :---: | :---: | :---: | :---: |
| A | 1 | 2 | 3 |
| B | 2 | $3 \frac{1}{2}$ | 8 |
| C | 6 | 9 | 18 |
| D | 4 | $5 \frac{1}{2}$ | 10 |
| E | 1 | $4 \frac{1}{2}$ | 5 |
| F | 4 | 4 | 10 |
| C | 5 | $6 \frac{1}{2}$ | 11 |
| H | 5 | 8 | 17 |
| I | 3 | $7 \frac{1}{2}$ | 9 |
| J | 3 | 9 | 9 |
| K | 4 | 4 | 4 |
| L | 1 | $5 \frac{1}{2}$ | 7 |
| M | 1 | 2 | 3 |
| N | 5 | $5 \frac{1}{2}$ | 9 |

Table 3:Time-Cost trade off data for the project activities

| Activity | Time |  | Cost |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Normal | Crash | Normal | Crash |
|  | 2 weeks | 1 week | $\$ 180,000$ | $\$ 280,000$ |
| B | 4 weeks | 2 weeks | $\$ 320,000$ | $\$ 420,000$ |
| C | 10 weeks | 7 weeks | $\$ 620,000$ | $\$ 860,000$ |
| D | 6 weeks | 4 weeks | $\$ 260,000$ | $\$ 340,000$ |
| E | 4 weeks | 3 weeks | $\$ 410,000$ | $\$ 570,000$ |
| F | 5 weeks | 3 weeks | $\$ 180,000$ | $\$ 260,000$ |
| C | 7 weeks | 4 weeks | $\$ 900,000$ | $\$ 1,020,000$ |
| H | 9 weeks | 6 weeks | $\$ 200,000$ | $\$ 380,000$ |
| I | 7 weeks | 5 weeks | $\$ 210,000$ | $\$ 270,000$ |
| 1 | 8 weeks | 6 weeks | $\$ 430,000$ | $\$ 490,000$ |
| K | 4 weeks | 3 weeks | $\$ 160,000$ | $\$ 200,000$ |
| L | 5 weeks | 3 weeks | $\$ 250,000$ | $\$ 350,000$ |
| M | 2 weeks | 1 week | $\$ 100,000$ | $\$ 200,000$ |
| N | 6 weeks | 3 weeks | $\$ 330,000$ | $\$ 510,000$ |

b. What is the probability of completing the project in 40 days?
c. What is the probability of meeting the deadline of 47 days?
d. Evaluate the option of working towards getting the bonus. What is your comment on this option? [5]
3. You are responsible for transporting four items on a limited space of 10 tons from Windhoek to Swakopmund. There are four different items that your company can transport between Windhoek and Swakopmund. Each item has a weight in tons, a net profit in thousands of dollars, and a total number of item that is available for shipping as shown in Table below. Use dynamic programming to determine how many of each item should be shipped to maximize profits.

| Item | Weight | Profit / Unit (\$) | Number Available |
| :--- | :--- | :--- | :--- |
| 1 | 1 | 3 | 6 |
| 2 | 4 | 9 | 1 |
| 3 | 3 | 8 | 2 |
| 4 | 2 | 5 | 2 |

4. You are responsible for transporting explosives through a network of towns and you have to minimize the travel distance between 1 and 7 . Use an appropriate method to determine the minimal distance you need to travel.
a. Give examples of how networks are applied to solve problems in mining? [6]

5. One representation of economic order quantity ( EOQ ) inventory model is;
$\mathrm{Q}=\sqrt[2]{2 C A / c}$
Where Q is the economic order quantity
C is the cost of placing an order
$A$ is the annual demand in units
c is the cost of holding one unit in stock for one year
Data relevant to component K used at a mine in 22 different sections include;
Purchase price: $\$ 15$ per 100,
Annual usage: 100,000 units ,
Cost of buying office: fixed 15,575 per annum and variable is $\$ 12$ per order,
Rent of warehouse: $\$ 3,000$ per annum,
Heating: \$700 per annum
Interest: $25 \%$ per annum, insurance $0.05 \%$ per annum based on total purchases, deterioration has been expressed as $1 \%$ per annum of all items purchased.

## Questions:

a. Calculate the EOQ for component K .
b. Calculate the percentage change in total annual variable costs relating to component K if the annual usage was: (i) 125,000 units, and (ii) 75,000 units.
c. Use the answers for question $b$ above to comment on the sensitivity of the variable costs to changes in the annual usage.
d. Describe two methods to help you modify your EOQ calculations if management decided that the expected total investment in stocks was $30 \%$ too high.

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(End of Exam)
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TABLE 1
Standard normal curve areas


| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 08264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | $0.86+3$ | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | $0.89+4$ | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.974 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.99 .45 | 0.9946 | 0.99 .48 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9969 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

## Queuing Model, Single Server Formulas

$$
\begin{aligned}
P_{o} & =\operatorname{Prob}\left[\begin{array}{l}
\text { system is } \\
\text { empty (idle) }
\end{array}\right]=1-\frac{\lambda}{\mu} \\
L_{q} & =\begin{array}{l}
\text { average number } \\
\text { in the queue }
\end{array}=\frac{\lambda^{2}}{\mu(\mu-\lambda)} \\
L & =\begin{array}{l}
\text { average number } \\
\text { in the system }
\end{array}=\frac{\lambda}{\mu-\lambda} \\
W_{q} & =\begin{array}{l}
\text { average time } \\
\text { in the queue }
\end{array}=\frac{\lambda}{\mu(\mu-\lambda)} \\
W & =\begin{array}{l}
\text { average time } \\
\text { in the system }
\end{array}=\frac{1}{\mu-\lambda}
\end{aligned}
$$

Note:
$\lambda$ is the arrival rate.
$\mu$ is the service rate.

